# APMA E4204: Functions of a Complex Variable

# Course Syllabus, Fall 2024

**Course Overview.** The main objective of this course is to provide an elementary introduction to applied complex analysis. The course include roughly the following main topics: (i) fundamental properties of the complex number system, elementary topology on the complex plane, and elementary properties of complex functions; (ii) differentiation on the complex plane, analyticity and its consequences; (iii) contour integrals and Cauchy's theorems; (iv) Taylor and Laurent series of complex functions; (v) singularities of functions and residue calculus; and (vi) selected applications in computing Fourier transforms, estimating zeros of complex functions, and related topics if time permits.

**Course Prerequisite.** Students are expected to: (a) have strong background in integral and differential calculus of one- and multi-variable functions; (b) be familiar with basic concepts of analysis (such as convergence of sequences and series of real numbers, and $\delta - \varepsilon$ characterization of continuity of real-valued functions).

**Class Meetings.** Monday and Wednesday 1:10 PM-2:25 PM @ 1024 Mudd.

**Instructor.** Xuenan Li (284A Mudd; xl3383@columbia.edu).

**Office Hours.** Monday 12 PM - 1PM @ Mudd 287 + Thursday 4 PM- 5PM @ Mudd 287 + Appointments

The following Thursday office hours will be moved to 3 PM - 4PM in my office Mudd 284 due to faculty meetings:

- September 12, September 26, October 10, November 14, December 12.

**TA's Office Hours.** Kavish Trivedi (kt3017@columbia.edu) Friday 1 PM - 2 PM @ Mudd 214; Wendi Lu (wl2931@columbia.edu) Tuesday 12 PM - 1 PM @ Mudd 287

**Textbooks.** The course is based on the following textbooks. All three textbooks can be found online (just Google the names) or in the library.

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Complex Variables: Introduction and Applications (recommended)
Second Edition
Mark J. Ablowitz and Athanassios S. Fokas
Cambridge University Press, 2003

Introduction to Complex Analysis (recommended)
Michael E. Taylor
American Mathematical Society, 2019

Complex Analysis
Elias M. Stein and Rami Shakarchi
Princeton University Press, 2003
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**Class Attendance.** Attendance is suggested at all class meetings, but not required. I will not take class attendance for credit.

**Homework.** There will be 11 homework sets, each based on 100 points. *The lowest score from your homework will be dropped automatically when calculating your final grade* (in other words, 10 of your highest homework scores will be used in calculating your final grade for the class).

Submission of homework will also be handled by Gradescope. There is a 4-day period for late homework submission. Homework solutions will be provided after the late homework submission deadline, so please hand in your homework before that deadline. *No submission is allowed after the solution is posted.*

**Exams.** There will be an in-class midterm exam, based on 100 points, and a final exam, also based on 100 points.

**Grading Policy.** The final grade will be weighted roughly as follows:

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Homework 30%, Midterm 30%, Final 40%
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**Course Webpage.** All the course material will be posted on the university teaching tool, the `Courseworks` system:

`https://courseworks.columbia.edu/`

**Important Dates.**

- 09/04/2024, First day of class for E4204
- 09/13/2024, Last day to "Drop the Class for Tuition Refund"
- 10/23/2024, Midterm review (a recorded video, no in-person class)
- 10/28/2024, In-class midterm, 75-minute exam
- 11/05/2024, Election day (no class)
- 11/14/2024, Complex Variables: Introduction and Applications/2023, Last day to "Drop or Change to Pass/Fail"
- 11/27/2024, Thanksgiving Break (no class)
- 12/09/2024, Last day of class for E4204
- ??/??/2024, Final exam for E4204 (TBD, a two-hour exam!)

**Academic Dishonesty.** Discussions and team works among students are encouraged in general. However, the work a student submits for grading, including homework and exams, must be his/her own work. Students who violate university rules on academic dishonesty are subject to disciplinary penalties, including the possibility of failing the course and/or dismissal from the University. Detailed information on academic integrity at Columbia University is available here:

`https://www.college.columbia.edu/academics/academicintegrity`

More explanation on dishonesty in academic work can be found here:

`https://www.college.columbia.edu/academics/academicdishonesty`

**Students with Disabilities.** Columbia University makes every effort to accommodate students with disabilities. If you require disability accommodations to attend the classes or the exams, please contact Columbia Disability Services at 212-854-2388. For more information, please visit:

`https://health.columbia.edu/content/disability-services`

Students with disabilities may be eligible for accommodations related to the administration of examinations. Here are more details:

`https://health.columbia.edu/services/testing-accommodations`

**Tentative Course Schedule.** Here is a tentative plan for the lectures.

- 09/04 (Wed): Course logistics and Intro to Complex System; HW #01
- 09/09 (Mon): Complex System;
- 09/11 (Wed): Elementary topology in complex system (I); HW #02
- 09/16 (Mon): Elementary topology in complex system (II) ;
- 09/18 (Wed): Elementary Complex Functions (I); HW #03
- 09/23 (Mon): Elementary Complex Functions (II);
- 09/25 (Wed): Continuity of Complex Functions (I); HW #04
- 09/30 (Mon): Continuity of Complex Functions (II);
- 10/02 (Wed): Complex Differentiation (I); HW #05
- 10/07 (Mon): Complex Differentiation (II);
- 10/09 (Wed): Complex Integration (I); HW #06
- 10/14 (Mon): Complex Integration (II);
- 10/16 (Wed): Cauchy's Integral Theorem (I); HW #07
- 10/21 (Mon): Cauchy's Integral Theorem (II);
- 10/23 (Wed): Midterm review (recorded video, no in-person class);
- 10/28 (Mon): In-Class Midterm;

- 10/30 (Wed): Cauchy's Derivative Theorem; HW #08
- 11/04 (Mon): Academic Holiday, No Class;
- 11/06 (Wed): Maximum Modulus Principle (I);
- 11/11 (Mon): Maximum Modulus Principle (II);
- 11/13 (Wed): Analytic Continuation Principle; HW #09
- 11/18 (Mon): Introduction to Power Series;
- 11/20 (Wed): Taylor and Laurent Series (I); HW #10
- 11/25 (Mon): Taylor and Laurent Series (II);
- 11/27 (Wed): Thanksgiving Holiday, No Class;
- 12/02 (Mon): Residue Calculus (I); HW #11
- 12/04 (Wed): Residue Calculus (II);
- 12/09 (Mon): Final review.
- 12/??: **Final Exam**, Time: TBD, Location: TBD.

This schedule is only tentative. Changes of the schedule will be announced in class. Homework will be posted online on the dates indicated.