APMA E4101: Dynamical systems

Course Syllabus, Spring 2025

Course Overview. This course provides an in-depth exploration of dynamical systems, a branch of mathematics and science that studies how systems change and evolve over time. Dynamical systems theory has wide-ranging applications in various fields, including physics, engineering, biology, economics, and environmental science. In this course, students will gain a solid foundation in the principles and techniques of dynamical systems analysis. The course include roughly the following main topics: (1) first-order ODE systems; (2) linear systems and stability; (3) nonlinear dynamics (phase space, limit cycles and bifurcations); (4) applications of dynamical systems; (5) a brief introduction to chaos.

Course Prerequisite. This is an advanced mathematical course, designed for upper level undergraduates and graduate students from the various programs in SEAS. Therefore, we will assume prior familiarity with calculus (including single variable and multi-variable calculus), elementary analysis (e.g. ϵ**-**δ **language**), a basic course in ordinary differential equations (**you must have learned how to solve ODE systems**), and linear algebra.

Class Meetings. Monday and Wednesday 8:40 AM-9:55 AM @ 702 HAMILTON

Instructor. Xuenan Li (Mudd 284 A; xl3383@columbia.edu).

Office Hours. Monday 10 - 11 AM and Thursday 4-5 PM in Mudd 287
+ Appointments via email

Textbooks. The course is based on the following textbooks. Both textbooks can be found online (just Google the names) or in the library.

`Steven H. Strogatz`
`Nonlinear Dynamics and Chaos: With Applications to Physics,`
`Biology, Chemistry, and Engineering (2nd edition)`

`Morris W. Hirsch, Stephen Smale, and Robert L. Devaney`
`Differential Equations, Dynamical Systems,`
`and an Introduction to Chaos (3rd edition)`

Class Attendance. Attendance is encouraged but not required at all class meetings.

Homework. There will be 10 homework sets. The lowest score from your homework will be dropped when calculating your final grade. Homework solutions will be provided a few days after each deadline. No homework will be accepted after the solutions are posted. Submission of homework will be handled by **Gradescope**.

Late submission of homework assignments without valid reasons will encounter late penalties (late penalty is 15% per day calculated automatically).

Exams. There will be an in-class midterm exam and a final exam in the final week.

Grading Policy. The final grade will be weighted roughly as follows:

`Homework 30%, Midterm 30%, Final 40%`

Course Webpage. All the course material will be posted on the university teaching tool, the `Courseworks` system:

`https://courseworks.columbia.edu/`

Important Dates.

- 01/22/2024, First day of class for E4101
- 01/31/2024, Last day to "Drop the Class for Tuition Refund"
- 03/10/2024, Midterm review
- 03/12/2024, In-class midterm
- 03/11/2024 – 03/15/2024, Spring recess (no class)
- 03/27/2024, Last day to "Drop or Change to Pass/Fail"
- 05/05/2024, Last day of class for E4101
- ??/??/2024, Final exam for E4101 (TBC but it is a two-hour exam!)

Academic Dishonesty. Discussions and team works among students are encouraged in general. However, the work a student submits for grading, including homework and exams, must be his/her own work. Students who violate university rules on academic dishonesty are subject to disciplinary penalties, including the possibility of failing the course and/or dismissal from the University. Detailed information on academic integrity at Columbia University is available here:

`https://www.college.columbia.edu/academics/academicintegrity`

More explanation on dishonesty in academic work can be found here:

`https://www.college.columbia.edu/academics/academicdishonesty`

Students with Disabilities. Columbia University makes every effort to accommodate students with disabilities. If you require disability accommodations to attend the classes or the exams, please contact Columbia Disability Services at 212-854-2388. For more information, please visit:

`https://health.columbia.edu/content/disability-services`

Students with disabilities may be eligible for accommodations related to the administration of examinations. Here are more details:

`https://health.columbia.edu/services/testing-accommodations`

Tentative Course Schedule. Here is a tentative plan for the lectures.

- 01/22 (Wed): Course overview and introduction to dynamical systems
- 01/27 (Mon): (**Review**) Examples of solving ODE IVPs and the Existence and Uniqueness Theorem of ODE systems; HW #01
- 01/29 (Wed): Proof of the Existence and Uniqueness Theorem using the Picard iteration;
- 02/03 (Mon): 1D flow: linear stability; HW #02
- 02/05 (Wed): 1D flow: bifurcation I;
- 02/10 (Mon): 1D flow: bifurcation II; HW #03
- 02/12 (Wed): 1D flow: bifurcation III (applications and examples);
- 02/17 (Mon): (**Review**) Linear Systems I: how to solve $X' = AX$ HW #04

- 02/19 (Wed): Linear Systems II: Phase portraits and classification of fixed points;
- 02/24 (Mon): Nonlinear 2D systems I (phase portraits, fixed points and null clines); HW #05
- 02/26 (Wed): Nonlinear 2D systems II (linear stability analysis);
- 03/03 (Mon): Nonlinear 2D systems III (Example: the Lotka-Volterra models of competition/ predator-prey);
- 03/05 (Wed): Hamilitonian systems: the nonlinear pendulum; HW #06
- 03/10 (Mon): Midterm review;
- 03/12 (Wed): In-Class Midterm;
- 03/17 (Mon): Spring recess, No Class;
- 03/19 (Wed): Spring recess, No Class;
- 03/24 (Mon): Reversible systems;
- 03/26 (Wed): Analysis of periodic harvesting: the Poincare' map; HW #07
- 03/31 (Mon): Index theory I;
- 04/02 (Wed): Index theory II;
- 04/07 (Mon): Limit Cycles I (definition and examples); HW #08
- 04/09 (Wed): Limit Cycles II (Poincare-Bendixson Theorem);
- 04/14 (Mon): Bifurcations in 2D systems;
- 04/16 (Wed): Limit Cycles III (Hopf bifurcations); HW #09
- 04/21 (Mon): Beyond 2D systems: higher-dimensional linear systems;
- 04/23 (Wed): Introduction to chaos: impossibility of chaos in 2D and Lorenz system I; HW #10
- 04/28 (Mon): Lorenz system II;
- 04/30 (Wed): Discrete Systems: The Lorenz Map;
- 05/05 (Mon): Final review.

- **Final Exam**, Time: TBD, Location: TBD.

This schedule is only tentative. Changes of the schedule will be announced in class. Homework will be posted online on the dates indicated. If the material is labeled with **Review**, it means that you should have learned in a prior course.